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Benefits of VisionGauge® Systems for Gage R&R Testing

A Gage Repeatability and Reproducibility study (Gage R&R) is a method to assess the repeatability and reproducibility of a measurement system.

Repeatability is the variation between successive measurements of the same part or trait by the same person using the same gage. In other words, how much variation do we see in measurements taken by the same person, on the same part, using the same tool?

Reproducibility is the difference in the average of the measurements made by different people using the same instrument when measuring the identical characteristic on the same part. In other words, how much variation do we see in measurements taken by different people on the same part using the same tool?

Manufacturers are continually striving to improve product and process quality and reduce part variation in their processes. The only way they have of doing this is by measuring the part variation. And the measured part variation is the sum of the process variation (which is what, in the end, interests manufacturers) and measurement system variation, which is what a Gage R&R study quantifies. So we can see the importance of properly quantifying the measurement system variation through a proper Gage R&R study.

And this is one of the areas where VisionGauge® systems really shine. The measurement instrument is of course an important component of the overall measurement system (which also includes workholding, the operators, etc…). And VisionGauge®, because it produces very repeatable measurements and operator-independent results, is ideal for minimizing the measurement system Gage R&R.

In this document, we review the technical details and implementation of a Gage R&R study more fully, we present sample results, and then we go over the benefits of VisionGauge® specifically in this context.

1. Gage R&R - Overview

A Gage R&R (repeatability and reproducibility) measures the amount of variability induced in measurements by the measurement system itself and compares it to the total variability observed to determine the viability of the measurement system.

There are several factors affecting a measurement system, including:

Measuring instruments: the gage or instrument itself

Fixturing / workholding: the repeatability and reproducibility of the fixturing / workholding that positions the part can have a large effect on a GR&R result

Operators: the ability and/or discipline of a person to follow the written or verbal instructions.

Specification: the measurement is reported against a specification or a reference value. The range or the engineering tolerance does not affect the measurement, but is an important factor in evaluating the viability of the measurement system.

Parts: some items are easier to be measured than others. A measurement system may be good for measuring steel block length but not for measuring rubber pieces, for example.

Etc...

There are two important aspects of a Gage R&R:

Repeatability: the variation between successive measurements of the same part or feature by the same person using the same gage. In other words, how much variation do we see in measurements taken by the same person, on the same part, using the same tool?

Reproducibility: the variation of the measurements made by different people using the same instrument when measuring the identical characteristic on the same part. In other words, how much variation do we see in measurements taken by different people on the same part using the same tool?

Gage R&R addresses only the *precision* of a measurement system. It is common to examine the *P/T* ratio which is the ratio of the precision of a measurement system to the (total) tolerance of the manufacturing process of which it is a part. If the P/T ratio is low, the impact on product quality of variation due to the measurement system is small. If the P/T ratio is larger, it means the measurement system is "eating up" a large fraction of the tolerance, in that the parts that do not have sufficient tolerance may be measured as acceptable by the measurement system.

Generally, a P/T ratio less than 0.1 (10%) indicates that the measurement system can reliably determine whether any given part meets the tolerance specification.

A P/T ratio between 0.1 (10%) and 0.3 (30%) indicates that the measurement system can adequately determine whether any given part meets the tolerance specification.

A P/T ratio greater than 0.3 (30%) suggests that unacceptable parts will be measured as acceptable (or vice-versa) by the measurement system, making the system inappropriate for the process for which it is being used.

2. Gage R&R - Calculations

Notes:

1. All calculations use a 5.15 sigma value (which corresponds to the limits that contain the central 99.0% of the area under a normal distribution curve)
2. Calculations are based on the X-Bar & R-Chart approach.

a) Repeatability Study - Equipment Variation (EV)

Single operator performs multiple trials
Provides a quick estimate of measurement capability

$$EV = \bar{R}K_1$$

Where: $\bar{R}$ is the average range: $\frac{1}{3}(\bar{R}_a + \bar{R}_b + \bar{R}_c)$

Trials	K_1
2	4.56
3	3.05

K_1 = 5.15/d_2, where d_2 is dependent on the number of trials, and the number of parts times the number of operators, which is assumed to be greater than 15

b) Reproducibility study - Appraiser Variation (AV)

Multiple operators perform multiple trials
Provides an estimate of measurement variances due to operator differences in practice.

$$AV = \sqrt{(\bar{R}_M K_2)^2 - \left(\frac{EV^2}{nr}\right)}$$

Operators	K_2
2	3.65
3	2.70

Where: $\bar{R}_M$ is range of the operator average (max $\bar{X}$ - min $\bar{X}$)
n is number of parts
r is number of trials

K_2 = 5.15/d_2, where d_2 is dependent on the number of operators

c) Gage R&R

Variation due to Repeatability and Reproducibility

$$R\&R = \sqrt{EV^2 + AV^2}$$

Parts	K_3
2	3.65
3	2.70
4	2.30
5	2.08
6	1.93
7	1.82
8	1.56
9	1.67
10	1.62

d) Part to Part

Part variation

$$PV = R_P K_3$$

Where R_P is the range of the part average

K_3 = 5.15/d_2, where d_2 is dependent on the number of parts

e) Total Variation

Variation due to parts and the measurement system

$$TV = \sqrt{(R\&R)^2 + (PV)^2}$$

3. Gage R&R - Instructions

You will need:

- 2 or 3 operators
- Between 2 and 10 sample parts
- Ten parts, three operators, and three trials are typical.

Setup:

- Randomly label each part 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
- Record operator names

Run the experiment:

1. Ask each operator to measure each part once using their normal process. Enter the results into the Gage R&R
2. Ask each operator to measure the parts again. Enter these results into the second row of the Gage R&R
3. Repeat for trial 3

Results:

The results will be presented in the table.

Characteristics of a good gage:

<10% Satisfactory

10%-30% May be satisfactory

30%-100% Unsatisfactory. Take corrective action

4. Gage R&R - Derivation of Formulas

a) Model

In this model, the kth measurement made by operator j on part i (y_{ijk}) is described by:

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$$

Where µ is an unknown constant representing the average measurement.
The α's represent random effects due to the different parts
The β's represent random effects due to different operators
(note that αβ represents effects due to different part/operator combinations)
Є represents random measurement errors.

The variance components associated with each random effect are σ_α^2, σ_β^2, $\sigma_{\alpha\beta}^2$, and σ^2. These quantify the variation in parts, operators, part/operator combinations, and measurement error, respectively.

The purpose of a gage R&R is to estimate these variance components, specifically σ^2 and $\sigma_\alpha^2 + \sigma_{\alpha\beta}^2$.

- σ^2 gives an estimate of the measurement errors for repeated measurements on a fixed part/operator combination, therefore σ gives value for the repeatability, $\hat{\sigma}_{\text{repeat.}}$.
- $\sigma_\alpha^2 + \sigma_{\alpha\beta}^2$ gives an estimate of the measurement errors for many operators making a single measurement on the same part, assuming that there is no error in repeated measurements. Estimating $\sqrt{\sigma_\alpha^2 + \sigma_\beta^2}$ gives a value for the reproducibility, $\hat{\sigma}_{\text{reprod.}}$.

b) Repeatability

σ can be estimated from the range of the repeated measurements on the ith part made by the jth operator, R_{ij}. If measurements are normally distributed then

$$R_{ij} = d_2\sigma$$

d_2 is a conversion constant based on 3 trials and >15 [parts*operators] obtained from table D_2, "Quality Control and Industrial Statistics".

Therefore the estimate of σ should be

$$\sigma = {}^{R_{ij}}\!/_{d_2}.$$

Averaging these over all part/operator combinations, and predicting 5.15 sigma gives

$$\hat{\boldsymbol{\sigma}}_{\mathbf{repeat.}} = \mathbf{K_1}\overline{\mathbf{R}} \quad , \qquad where\ \mathrm{K}_1 = {}^{5.15}\!/_{\mathrm{d}_2}$$

c) Reproducibility

For a fixed part, the means of the operators differ by

$$\beta_j + \alpha\beta_{ij} + \bar{\epsilon}_{ijk}$$

$\bar{\epsilon}_{ijk}$ is an average of [trials (n)*operators (r)], nr, measurements. (This average will have a variance of σ^2/nr)

Which has a variance of

$$\sigma_\beta^2 + \sigma_{\alpha\beta}^2 + {\sigma^2}/{nr}$$

The range of the means (R_M) of part i can then be estimated by

$$R_{Mi} = d_2\sqrt{\sigma_\beta^2 + \sigma_{\alpha\beta}^2 + {\sigma^2}/{nr}}$$

d_2 is a conversion constant based on 3 operators obtained from table D_2, "Quality Control and Industrial Statistics".

Averaging this over all parts, and predicting 5.15 sigma gives

$$\sqrt{\sigma_\beta^2 + \sigma_{\alpha\beta}^2 + {\sigma^2}/{nr}} = K_2\bar{R}_M, \ \ where\ K_2 = {5.15}/{d_2}$$

Combining this with the information from section 2,

$$(K_2\bar{R}_M)^2 = \sigma_\beta^2 + \sigma_{\alpha\beta}^2 + {(K_1\bar{R})^2}/{nr}$$

So

$$\sqrt{\sigma_\beta^2 + \sigma_{\alpha\beta}^2} = \sqrt{(K_2\bar{R}_M)^2 - {(K_1\bar{R})^2}/{nr}}$$

Or

$$\boldsymbol{\hat{\sigma}_{repeat.}} = \sqrt{\boldsymbol{(K_2\bar{R}_M)^2 - {(K_1\bar{R})^2}/{nr}}}$$

5. Example of Gage R&R Results

The above presents the steps involved in a Gage R&R study, as well as the underlying rationale. The reader can also refer to ASTM F1469 ("Standard Guide for Conducting a Repeatability and Reproducibility Study on Test Equipment for Nondestructive Testing") for complete implementation details.

Below are representative sample results of a Gage R&R study. All the measurements for all the parts & operators are presented as well as the per-part, per-operator and overall averages and ranges.

The most import result is the overall Gage R&R (as a percentage of tolerance). In this example, it is 1.5922. Remember from above that <10% is satisfactory, 10%-30% may be satisfactory and 30%-100% is unsatisfactory and that corrective action is required.

Appraiser Name	Trial #	Part										Average
		1	2	3	4	5	6	7	8	9	10	
Operator A: Katie	1	28.583	28.575	28.415	28.533	28.662	28.567	28.493	28.532	28.601	28.488	28.5449
	2	28.587	28.575	28.416	28.535	28.663	28.568	28.492	28.531	28.6	28.488	28.5455
	3	28.586	28.575	28.415	28.535	28.662	28.568	28.492	28.533	28.598	28.491	28.5455
	Average	28.585	28.575	28.415	28.534	28.662	28.568	28.492	28.532	28.600	28.489	28.545
	Range	0.004	0.000	0.001	0.002	0.001	0.001	0.001	0.002	0.003	0.003	0.0018
Operator B: Rick	1	28.583	28.573	28.416	28.537	28.665	28.563	28.494	28.534	28.599	28.491	28.5455
	2	28.586	28.576	28.416	28.535	28.667	28.566	28.493	28.533	28.599	28.49	28.5461
	3	28.587	28.574	28.415	28.535	28.662	28.567	28.495	28.533	28.602	28.492	28.5462
	Average	28.585	28.574	28.416	28.536	28.665	28.565	28.494	28.533	28.600	28.491	28.546
	Range	0.004	0.003	0.001	0.002	0.005	0.004	0.002	0.001	0.003	0.002	0.0027
Operator C: Patrick	1	28.583	28.576	28.413	28.532	28.665	28.566	28.493	28.531	28.6	28.486	28.5445
	2	28.586	28.576	28.416	28.537	28.661	28.567	28.493	28.532	28.596	28.489	28.5453
	3	28.587	28.576	28.416	28.537	28.662	28.567	28.492	28.536	28.601	28.491	28.5465
	Average	28.585	28.576	28.415	28.535	28.663	28.567	28.493	28.533	28.599	28.489	28.545
	Range	0.004	0.000	0.003	0.005	0.004	0.001	0.001	0.005	0.005	0.005	0.0033
	Part Average	28.585	28.575	28.415	28.535	28.663	28.567	28.493	28.533	28.600	28.490	28.546

	Value	% of Tolerance	%of Process Variation
Repeatability: Equipment Variation (EV)	0.0079	1.5818	1.9691
Reproducibility: Appraiser Variation (AV)	0.0009	0.1817	0.2261
Overall GR&R:	0.0080	1.5922	1.9820

6. Benefits of VisionGauge® Systems for Gage R&R Studies and Reducing Process Variability

Manufacturers are always working to improve product and process quality and reduce part variation in their processes. To do this, they measure part variation, and properly quantifying and minimizing the variation due to the overall measurement system is key. This will allow manufacturers to know the process variation, which is what they are working to quantify and minimize.

An overall measurement system is comprised of a number of components, including workholding (e.g. poor workholding can definitely degrade measurements), operators as well as a number of other factors. And, of course, one of the primary components of a measurement system is the measurement instrument itself.

VisionGauge® systems are ideally suited for the task of carrying out Gage R&R studies as they have shown themselves, time & time again, to be extremely repeatable and to produce operator-independent results. These are huge benefits for any instrument to be used for a Gage R&R study. The fact that VisionGauge® is also very fast and easy to use is also very nice, but these aren't advantages that will lead to an improved Gage R&R.

Note that in this document we've focused on so-called Type 2 Gage R&R studies. Note however that VisionGauge® systems are equally well suited to Type 1 Gage R&R studies (that are generally simpler and less involved) as well as Attribute Gage R&R studies (that are more concerned with part attributes, such as "Pass" and "Fail").

While VisionGauge® does not in itself constitute an entire measurement system, it is without a doubt an excellent tool to minimize Gage R&R results and thus to reduce process variability.

For more information about VisionGauge® measurement systems and software, visit https://www.visionxinc.com